SECTION B4.5

PLASTIC BENDING

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Concluded)

B4.5.0 Plastic Analysis of Beams

Introduction

The conventional flexure formula f=Mc/I is correct only if the maximum fiber stress is within the proportional limit. In the plastic range, the assumption that plane sections remain plane is valid while the stress corresponds with the stress-strain relationship of the material.

This section provides a method of approximating the true stress which depends upon the shape and the material properties. It is noted that deflection requirements are investigated when using this method since large deflections are possible while showing adequate structural strength.

The method outlined in this section is not applicable if the member is subjected to high fluctuating loads.

The following glossary is given for convenience:

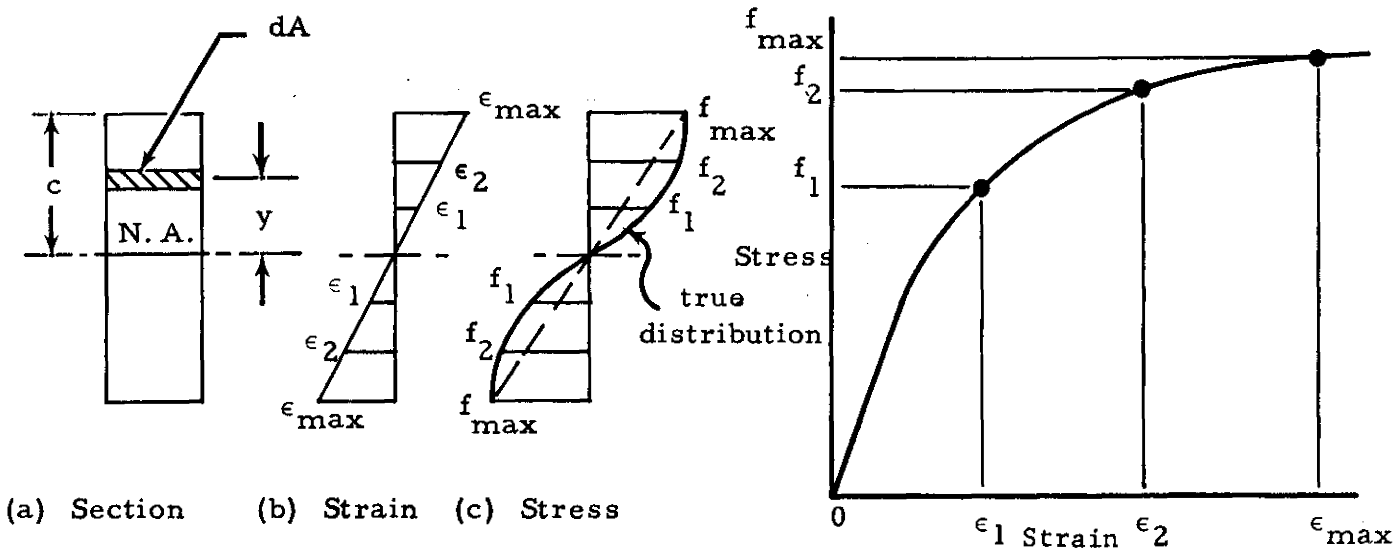
Simple bending: This condition occurs when the resultant applied moment vector is parallel to a principal axis.

Complex bending: This condition occurs when the resultant applied moment vector is not parallel to a principal axis.

Development of the Theory

A rectangular cross section will be used in this development; any other symmetrical cross section would yield the same results.

B 4.5.0 Plastic Analysis of Beams (Cont'd)



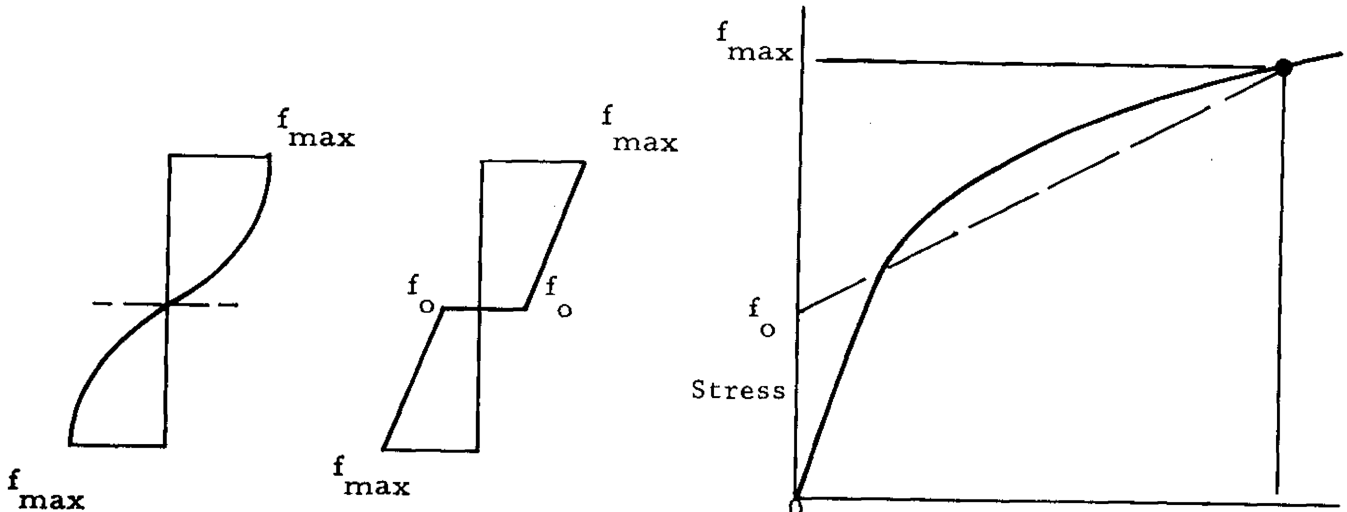
(a) Section (b) Strain (c) Stress (d) Stress-Strain Curve (Tension and Compression)

Figure B4.5.0-1

Since the bending moment of the true stress distribution about the neutral axis is greater than that of a linear Mc/I distribution as used in the elastic range, a trapezoidal stress distribution is used to approximate the true stress distribution. f_{max} may be defined as a yield stress, a buckling stress, an ultimate stress or any other stress level above the proportional limit.

Let f_o be a fictitious stress at zero strain in the trapezoidal stress distribution as shown in Figure B4.5.0-2. The value of f_o may be determined by integrating graphically the moment of (not the area of) the true stress distribution and equating this to the bending moment of the trapezoidal stress distribution.

B 4.5.0 Plastic Analysis of Beams (Cont'd)



(a) True Stress (b) Trapezoidal Stress (c) Stress Strain Curve

Figure B4.5.0-2

M_{b_a} = Allowable bending moment of the true stress distribution for a particular cross-section at a prescribed maximum stress level.

$F_b = \frac{M_{b_a} c}{I}$ = Fictitious allowable $\frac{Mc}{I}$ stress or the bending modulus of rupture for a particular cross-section at a prescribed maximum stress level.

$M_{b_t} = \frac{I}{c} f_{max} + (2Q - \frac{I}{c}) f_o$ = The bending moment of a trapezoidal stress distribution that is equivalent to M_{b_a}

Where for symmetrical sections,

$I = \int_{-c}^{c} y^2 \, dA$ (moment of inertia)

$Q = \int_{0}^{c} y \, dA$ (static moment of cross-section)

c = Distance from centroidal axis to the extreme fiber

B 4.5.0 Plastic Analysis of Beams (Cont'd)

Therefore, from $F_b = \frac{M_{b_t} c}{I}$ we obtain:

$$F_b = f_{max} + (k-1)f_o \qquad (4.5.0\text{-}1)$$

Where,

$$k = \frac{2Qc}{I} \qquad (4.5.0\text{-}2)$$

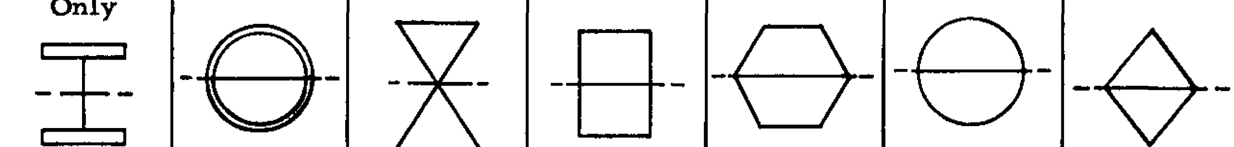
Two types of figures are presented for plastic bending analysis. One type presents Bending Modulus of Rupture Curves for Symmetrical Sections at yield and ultimate (Reference Section B4.5.4). The other type presents Plastic Bending Curves (Reference Section B4.5.5) which are necessary for the following:

1) Limiting stress other than yield or ultimate.

2) Tension and compression stress strain curves which are significantly different.

3) Unsymmetrical cross-section.

Figure B4.5.0-3 shows values of k for various symmetrical cross-sections. Generally, validity of the approach has been shown only for typical aircraft sections which are geometric thin sections.



Flanges Only	Thin Tube	Hourglass	Rectangle	Hexagon	Solid Round	Diamond
$k = 1.0$	$k = 1.27$	$k = 1.33$	$k = 1.5$	$k = 1.6$	$k = 1.7$	$k = 2.0$

SHAPE FACTOR

Figure B4.5.0-3

B4.5.1 Analysis Procedure When Tension and Compression Stress-Strain Curves Coincide.

B4.5.1.1 Simple Bending about a Principal Axis---Symmetrical Sections.

The procedure is as follows:

1. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

2. For yield or ultimate limiting stress, use the Bending Modulus of Rupture Curves (F_b vs. k - see section B4.5.5 for index) to determine F_b.

3. For a limiting stress other than yield or ultimate, use the Plastic Bending Curves (See section B4.5.6 for index). Locate the limiting stress on the stress-strain (or k=1) curve and move directly up to the appropriate k curve to read F_b for the same strain.

4. F_b from Step 3 and f_b, the calculated Mc/I stress, may be used in determining the bending stress ratio for combined stresses and the margin of safety for pure bending as follows:

$$R_b = \frac{f_b}{F_b} \qquad (4.5.1.1\text{-}1)$$

$$M.S. = \frac{1}{R_b(S.F.)} - 1 \qquad (4.5.1.1\text{-}2)$$

Where: S.F. is the appropriate (yield or ult) safety factor

B4.5.1.2 Simple Bending about a Principal Axis---Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

This procedure also applies to any section with bending about one of its principal axes.

B4.5.1.2 Simple Bending about a Principal Axis---Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending. (Cont'd)

The procedure is as follows:

1. Divide the section into two parts, (1) and (2), on either side of the principal axis (not the axis of symmetry) similar to that shown in Figure B4.5.1.2-1.

2. Calculate:

$$k_1 = \frac{Q_1 c_1}{I_1} \qquad (4.5.1.2\text{-}1)$$

where I_1 is the moment of inertia of part (1) only about the principal axis of the entire cross-section. This would be identical to k of a section made up of part (1) and its mirror image. Figure B4.5.0-3 may be used where part (1) and its mirror image form one of the sections shown.

3. Calculate similarly:

$$k_2 = \frac{Q_2 c_2}{I_2} \qquad (4.5.1.2\text{-}2)$$

4. Assuming part (1) is critical in yield (or crippling, ultimate, etc.) use the Plastic Bending Curves and locate this stress on the stress-strain (or k=1) curve. Move directly up to the appropriate k curve and read F_{b_1} for the same strain.

5. Read the strain ϵ_1.

6. Calculate:

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \qquad (4.5.1.2\text{-}3)$$

7. Locate ϵ_2 on the strain scale and move directly up to the appropriate k curve to read F_{b_2}

8. Calculate M_{b_a} by

$$M_{b_a} = \frac{F_{b1} I_1}{c_1} + \frac{F_{b2} I_2}{c_2} \qquad (4.5.1.2\text{-}4)$$

B 4.5.1.2 Simple bending about a principal axis--unsymmetrical sections with an axis of symmetry perpendicular to the axis of bending. (Cont'd)

9. M_{b_a} must be used in determining the moment ratio for bending and and the margin of safety for pure bending as follows:

$$R_b = \frac{M}{M_{b_a}} \qquad (4.5.1.2\text{-}5)$$

$$M.S. = \frac{1}{R_b \quad (S.F.)} - 1 \qquad (4.5.1.2\text{-}6)$$

Where: S.F. is the appropriate (yield or ult) safety factor

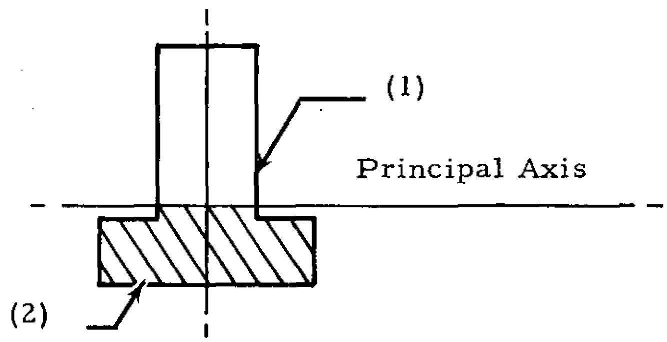
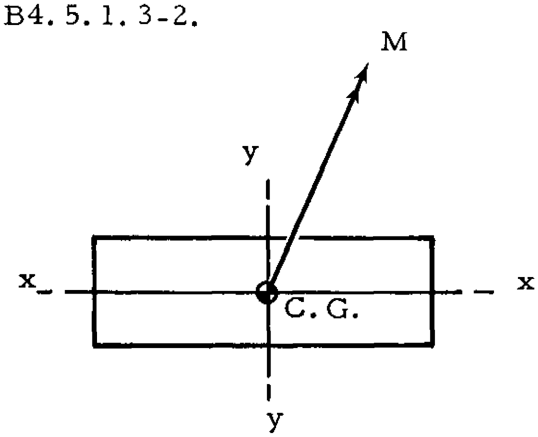


Figure B4.5.1.2-1

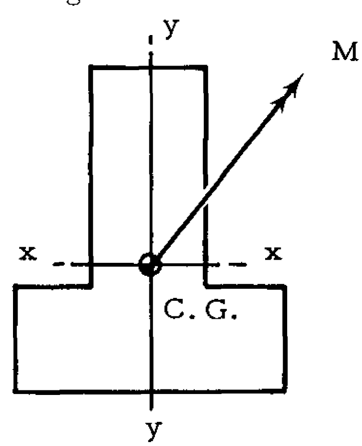
B4.5.1.3 Complex Bending--Symmetrical Sections; Also Unsymmetrical Sections with One Axis of Symmetry.

This condition occurs when the resultant applied moment vector is not parallel to a principal axis as shown in Figures B4.5.1.3-1 or B4.5.1.3-2.



Symmetrical Section

Figure B4.5.1.3-1



Unsymmetrical Section with one axis of symmetry

Figure B4.5.1.3-2

B 4.5.1.3 Complex Bending-- Symmetrical Sections; also Unsymmetrical Sections with One Axis of Symmetry.

The procedure is as follows (this procedure is always conservative and may be very conservative for some cross-sectional shapes):

1. Obtain M_x and M_y, the components of M with respect to the principal axes.

2. Follow the procedure outlined in Section B4.5.1.1 or B4.5.1.2 to determine R_{b_x} and R_{b_y}

3. $R_b = R_{b_x} + R_{b_y}$ (4.5.1.3-1)

4. For pure bending,

$$M.S. = \frac{1}{R_b\ (S.F.)} - 1 \qquad (4.5.1.3\text{-}2)$$

Where:
S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.4 Complex Bending--Unsymmetrical Sections with No Axis of Symmetry

This condition occurs when the resultant applied moment vector is not parallel to a principal axis as shown in Figure B4.5.1.4-1.

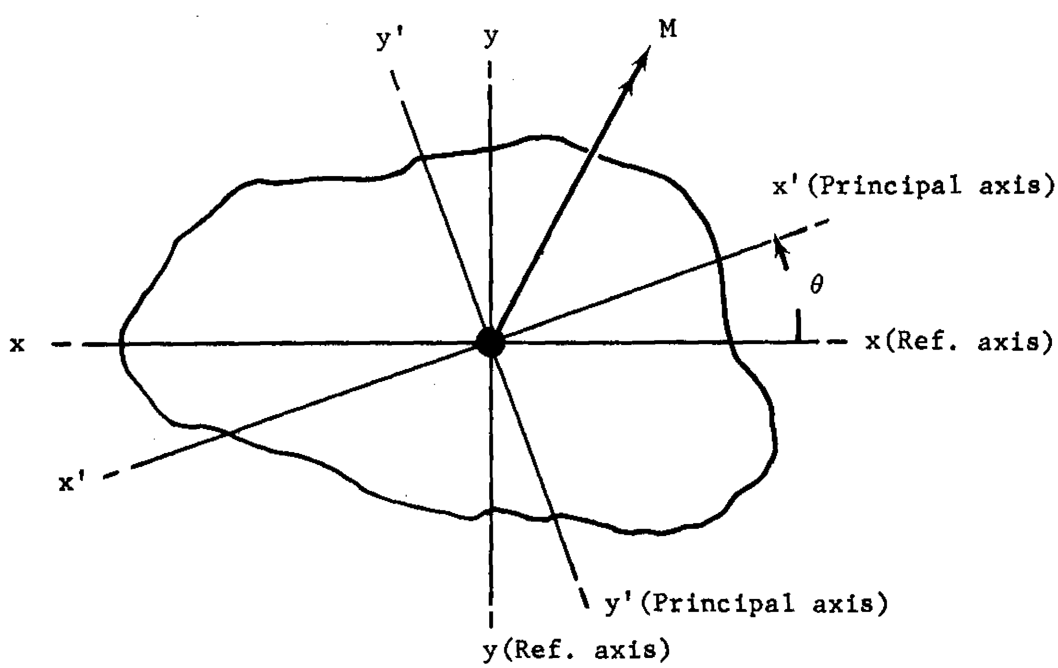


Figure B4.5.1.4-1

B 4.5.1.4 Complex Bending-- Unsymmetrical Sections with no Axis of Symmetry.

The procedure is as follows:

1. Determine the principal axes by the equation

$$\tan 2\theta = \frac{2 I_{xy}}{I_y - I_x} \qquad (4.5.1.4\text{-}1)$$

where x and y are centroidal axes and

$$I_x = \int y^2 \, dA \qquad \text{(Moment of inertia)}$$

$$I_y = \int x^2 \, dA \qquad \text{(Moment of inertia)}$$

$$I_{xy} = \int xy \, dA \qquad \text{(Product of inertia)}$$

2. Obtain $M_{x'}$ and $M_{y'}$, the components of M with respect to the principal axes.

3. Follow the procedure outlined in Section B4.5.1.2 to determine $R_{b_{x'}}$ and $R_{b_{y'}}$

4. The stress ratio for complex bending is

$$R_b = R_{b_{x'}} + R_{b_{y'}} \qquad (4.5.1.4\text{-}2)$$

5. For pure complex bending, the margin of safety is

$$M.S. = \frac{1}{R_b(S.F.)} - 1 \qquad (4.5.1.4\text{-}3)$$

where:

S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.5 Shear Flow for Simple Bending About a Principal Axis--Symmetrical Sections

When plastic bending occurs, the classical formula SQ/I is correct only for cross-sections with k=1.0. For k greater than 1.0, SQ/I is unconservative. The derivation of a correction factor β SQ/I is as follows for an ultimate strength assessment:

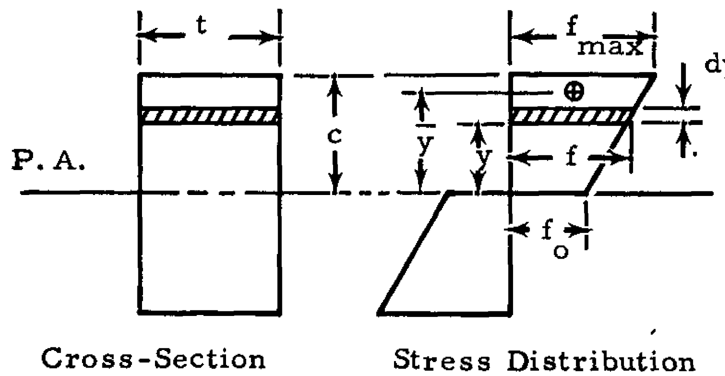


Figure B4.5.1.5-1

Let P = load on cross-section between y and c

$$P = \int_y^c tf dy$$

From the geometry of Figure B4.5.1.5-1

$$f = f_o + (f_{max} - f_o)\frac{y}{c}$$

$$\therefore P = \int_y^c f_o t\,dy + \int_y^c (f_{max} - f_o)\frac{ty}{c}\,dy$$

Let A= area of cross-section between y and c

$$A = \int_y^c t\,dy \quad \text{then,}$$

B 4.5.1.5 Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

$P = f_o A + (f_{max} - f_o) \frac{A\bar{y}}{c}$ and since $Q = A\bar{y}$,

$$P = f_{max} \left(\frac{Q}{c}\right) + f_o \left(A - \frac{Q}{c}\right) \qquad \text{(a)}$$

From Equation (4.5.0-1)

$$F_b = f_{max} + f_o (k - 1) = \frac{Mc}{I} \text{ then,}$$

$$\frac{c}{I} \frac{dM}{dx} = \frac{df_{max}}{dx} + \frac{df_o}{dx} (k - 1)$$

Since, by definition, $S = dM/dx$ and $q = dP/dx$,

$$\frac{c}{I} (S) = \frac{df_{max}}{dx} \left[1 + \frac{df_o}{df_{max}} (k - 1)\right] \text{ and from (a),} \qquad \text{(b)}$$

$$q = \frac{df_{max}}{dx} \frac{Q}{c} + \frac{df_o}{df_{max}} \frac{df_{max}}{dx} \left(A - \frac{Q}{c}\right)$$

Let $\lambda = df_o/df_{max}$ and $\beta = \frac{1 + \lambda (c/\bar{y} - 1)}{1 + \lambda (k - 1)}$ then,

$$q = \frac{df_{max}}{dx} \left[\frac{Q}{c} + \lambda \left(A - \frac{Q}{c}\right)\right] \qquad \text{(c)}$$

But since from (b): $df_{max}/dx = \frac{Sc}{I} / \left[1 + \lambda (k - 1)\right]$ (d)

and substituting (d) into (c) we have

$$q = \left(\frac{Sc}{I}\right)\left(\frac{Q}{c}\right) \frac{\left[1 + \lambda (Ac/Q - 1)\right]}{\left[1 + \lambda (k - 1)\right]}$$

Since $Q = A\bar{y}$,

$$q = \beta \frac{SQ}{I}$$

B 4.5.1.5 Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

The method outlined below shows how to correct SQ/I and calculate the margin of safety for simple plastic bending about a principal axis of a symmetrical section.

The procedure is as follows:

1. Determine Mc/I

2. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

3. Refer to the applicable Plastic Bending Curves and locate Mc/I on the F_b scale. Move across to the appropriate k curve to read the true strain, ϵ.

4. By use of the stress-strain (or k = 1) curve and the f_o curve, determine the rate of change of f_o with respect to f at the true strain ϵ, which would be expressed as

$$\lambda = \frac{df_o}{df} = \frac{df_o/d\epsilon}{df/d\epsilon} \qquad (4.5.1.5\text{-}1)$$

5. To determine the shear flow at any point on a cross-section, determine the distance from the principal axis to the centroid of the area outside of the point in question as shown in Figure B4.5.1.5-1. This is defined as $\overline{y}$.

6. Determine the shear flow at distance "a" from the principal axis by

$$q_a = \beta \frac{SQ_a}{I} \qquad (4.5.1.5\text{-}2)$$

where,

$$\beta = \frac{1 + \lambda\ (c/\overline{y} - 1)}{1 + \lambda\ (k - 1)} \qquad (4.5.1.5\text{-}3)$$

$$Q_a = \int_a^c y\, dA \qquad (4.5.1.5\text{-}4)$$

B 4.5.1.5. Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

7. Calculate $f_{s_{max}} = (f_s)_{a=0} = (q/t)_{a=0}$ and the stress ratio is

$$R_s = \frac{f_{s_{max}}}{F_s} \qquad (4.5.1.5\text{-}5)$$

8. For the margin of safety with pure bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{R_b^2 + R_s^2}} - 1 \qquad (4.5.1.5\text{-}6)$$

9. For the margin of safety with axial load, bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.5.1.5\text{-}7)$$

where,

$$R_a = \frac{fa}{Fa}$$

S. F. is the appropriate (yield or ult) safety factor.

B4.5.1.6 Shear Flow for Simple Bending About a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending

A procedure similar to that shown in Section B4.5.1.5 for Figure B4.5.1.6-1 is as follows:

1. Divide the section into two parts, (1) and (2), on either side of the principal axis (not the axis of symmetry) similar to that shown in Figure B4.5.1.2-1.

2. Calculate:

$$k_1 = \frac{Q_1 c_1}{I_1} \qquad (4.5.1.2\text{-}1)$$

where I_1 is the moment of inertia of part (1) only about the principal axis of the entire cross-section. This would be identical to k of a section made up of part (1) and its mirror image. Figure B4.5.0-3 may be used where part (1) and its mirror image form one of the sections shown.

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending.

3. Calculate similarly:

$$k_2 = \frac{Q_2 c_2}{I_2} \qquad (4.5.1.2\text{-}2)$$

4. Calculate $\frac{Mc_1}{I}$ and $\frac{Mc_2}{I}$

5. Refer to the applicable Plastic Bending Curves and locate $\frac{Mc_1}{I}$ on the F_b scale. Move across to the appropriate k (use k_1) curve to read the true strain ϵ_1.

6. Find ϵ_2, similarly, or by $\epsilon_2 = \frac{\epsilon_1 c_2}{c_1}$ (4.5.1.2-3)

7. By use of the stress-strain (or k = 1) curve and the f_o curve, determine the rate of change of f_o with respect to f for the true strain ϵ_1 which would be expressed as

$$\lambda_1 = \left(\frac{df_o}{df}\right)_1 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_1 \qquad (4.5.1.6\text{-}1)$$

8. Calculate similarly:

$$\lambda_2 = \left(\frac{df_o}{df}\right)_2 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_2 \qquad (4.5.1.6\text{-}2)$$

9. To determine the shear flow at any point on a cross-section, determine the distance from the principal axis to the centroid of the area above or below the point in question as shown in Figure B4.5.1.6-1. This is defined as $\bar{y}_a$ or $\bar{y}_b$.

10. Determine the shear flow in part (1) at distance "a" from the neutral axis by

$$q_a = \beta_a \left(\frac{SQ_a}{I}\right) \qquad (4.5.1.6\text{-}3)$$

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending.

where,

$$\beta_a = \frac{1 + \lambda_1 \left(\frac{c_1}{\bar{\bar{y}}_a} - 1 \right)}{1 + \lambda_1 (k_1 - 1)} \qquad (4.5.1.6\text{-}4)$$

$$Q_a = \int_a^{c_1} y dA \qquad (4.5.1.6\text{-}5)$$

11. Determine the shear flow in part (2) at distance "b" from the neutral axis by

$$q_b = \beta_b \left(\frac{SQ_b}{I} \right) \qquad (4.5.1.6\text{-}6)$$

where,

$$\beta_b = \frac{1 + \lambda_2 \left(\frac{c_2}{\bar{\bar{y}}_b} - 1 \right)}{1 + \lambda_2 (k_2 - 1)} \qquad (4.5.1.6\text{-}7)$$

$$Q_b = \int_b^{c_2} y dA \qquad (4.5.1.6\text{-}8)$$

12. For the shear flow at the principal axis, calculate q using both parts of the cross-section and use the larger. A rigorous analysis could be made so that shear flow calculations at the neutral axis would result in the same value regardless of which side was used in the calculations. This would involve determining the amount of transverse shear distributed to each side of the neutral axis. If the bending moment is developed entirely from external shear, then the shear distributed on each side is proportional to the corresponding moments and Equations (4.5.1.6-3) and (4.5.1.6-6) become as follows:

$$q_a = \beta_a \frac{S_1 Q_a}{I_1} \quad \text{where } S_1 = \frac{M_1}{M_1 + M_2} S, \; M_1 = \frac{F_{b_1} I_1}{C_1}$$

(See B4.5.1.2)

B 4.5.1.6 <u>Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending</u>.

$$q_b = \beta_b \frac{S_2 Q_b}{I_2} \quad \text{where } S_2 = \frac{M_2}{M_1 + M_2} S, \; M_2 = F_{b_2} \frac{I_2}{C_2}$$

(See B4.5.1.2)

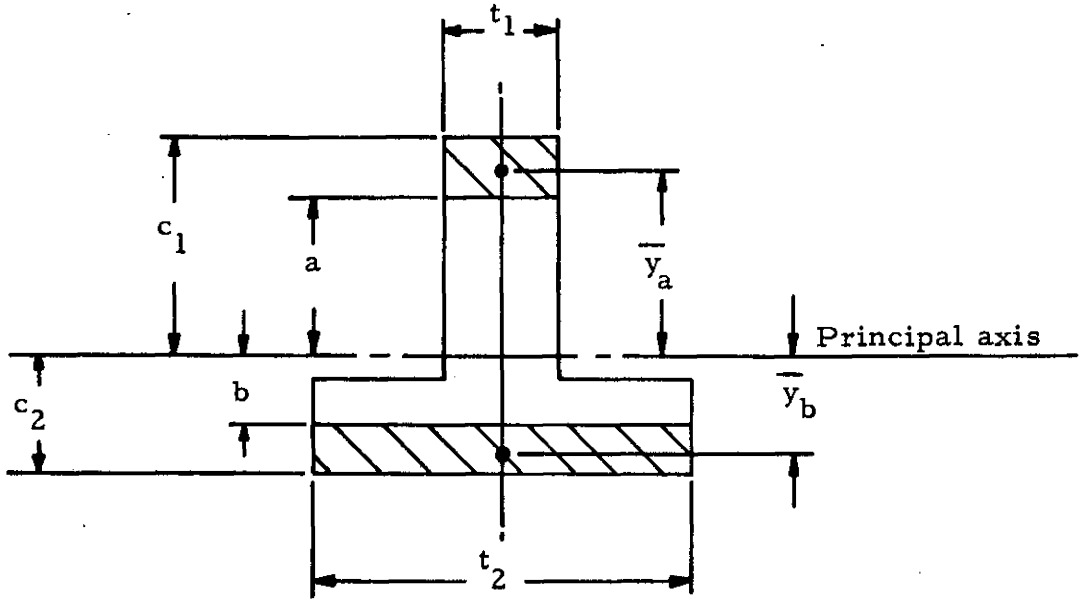


Figure B4.5.1.6-1

13. Calculate $f_{s_a} = \frac{q_a}{t_1}$ and $f_{s_b} = \frac{q_b}{t_2}$ to obtain the shear ratios:

$$R_{s_a} = \frac{f_{s_a}}{F_s} \qquad (4.5.1.6\text{-}9)$$

$$R_{s_b} = \frac{f_{s_b}}{F_s} \qquad (4.5.1.6\text{-}10)$$

14. For the margin of safety with pure bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{R_b^2 + R_s^2}} - 1 \qquad (4.5.1.6\text{-}11)$$

where:

S.F. is the appropriate (yield or ult) safety factor.

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending. (Cont'd)

15. For the margin of safety with axial load, bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.5.1.6\text{-}12)$$

where:

$$R_a = \frac{f_a}{F_a}$$

S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.7 Shear Flow for Complex Bending--Any Cross-Section

The procedure is as follows:

1. Determine the principal axes by inspection or, if necessary, by Equation (4.5.1.4-1).

2. Obtain $S_{x'}$ and $S_{y'}$, the components of S with respect to the principal axes. The principal axes are denoted by x' and y' as indicated in Figure B4.5.1.4-1.

3. Follow the same procedure of Section B4.5.1.6 about both principal axes to obtain the shear stresses $f_{s_{x'}}$ and $f_{s_{y'}}$ at a prescribed point.

4. The shear stress ratios at this point are

$$R_{s_{x'}} = \frac{f_{s_{x'}}}{F_s} \qquad (4.5.1.7\text{-}1)$$

$$R_{s_{y'}} = \frac{f_{s_{y'}}}{F_s} \qquad (4.5.1.7\text{-}2)$$

5. For the margin of safety with complex bending and shear use

$$M.S. = \frac{1}{S.F.\sqrt{R_b^2 + R_s^2}} - 1 \qquad (4.1.5.7\text{-}3)$$

B 4.5.1.7. Shear Flow for Complex Bending-- Any Cross-Section. (Cont'd)

where,

$$R_s = \sqrt{R_{s_{x'}}^{2} + R_{s_{y'}}^{2}} \qquad (4.1.5.7\text{-}4)$$

S. F. is the appropriate (yield or ult.) safety factor.

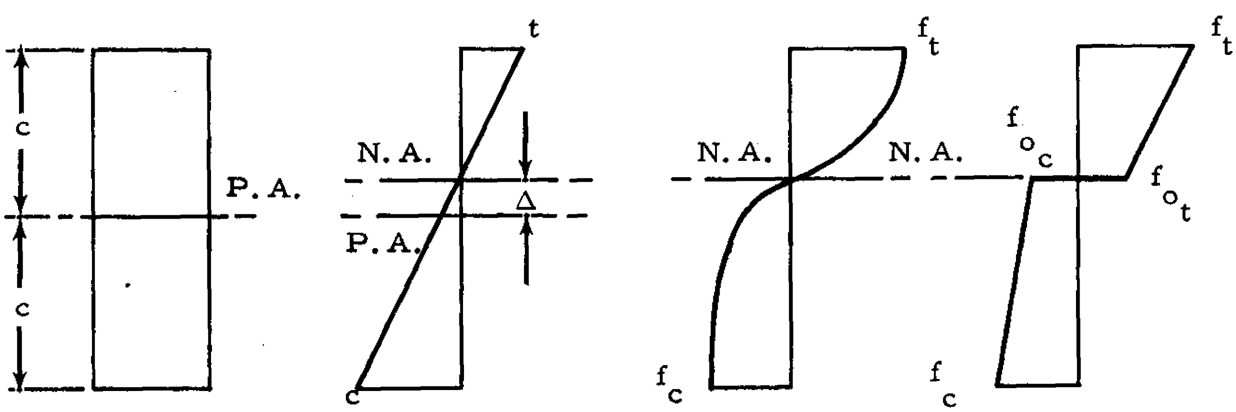
6. For the margin of safety with axial load, complex bending and shear use

$$M.S. = \frac{1}{S.F.\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.1.5.7\text{-}5)$$

B4.5.2 Analysis Procedure When Tension and Compression Stress-Strain Curves Differ Significantly

This may occur in materials such as the AISI 301 strainless steels in the longitudinal grain direction where the tension stress-strain curve is higher than the compression curve.

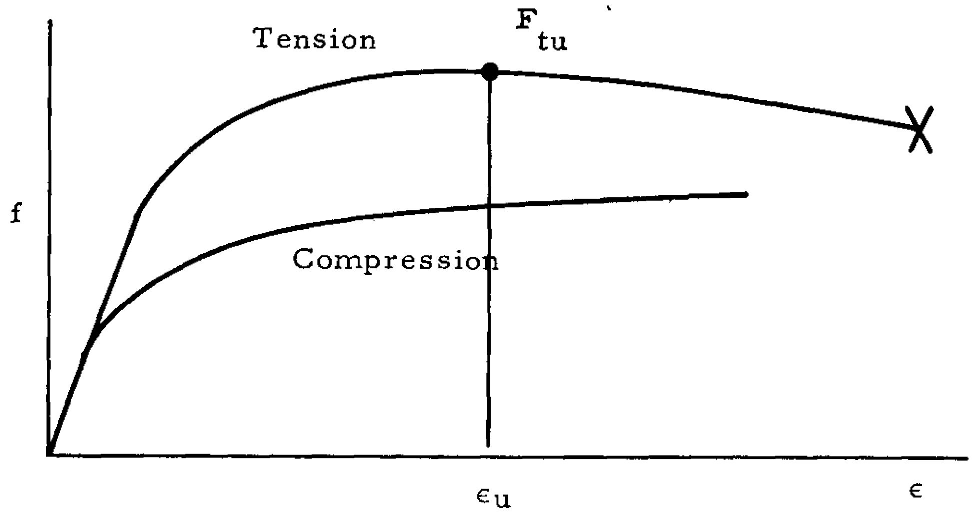
B4.5.2.1 Simple Bending about a Principal Axis--Symmetrical Sections



(a) Section (b) Strain (c) True Stress (d) Trapezoidal Stress

Figure B4.5.2.1-1

B 4.5.2.1 Simple Bending about a Principal Axis-- Symmetrical Sections. (Cont'd)



(e) Stress-Strain Curves
Figure B4.5.2.1-1 (cont.)

The procedure for a symmetrical section such as that in Figure B4.5.2.1-1 is as follows:

1. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

2. For yield or ultimate limiting stress, use the Bending Modulus of Rupture Curves to determine F_b. The correction for shifting the neutral axis away from the principal axis by Δ has been taken into account in the development of the curves. Note Step 8.

3. For a limiting stress (or strain) other than yield or ultimate, use the Plastic Bending Curves. Locate the limiting stress on the appropriate (tension or compression) stress-strain (or k=1) curve and call this f_1 at ϵ_1 with its trapezoidal intercept f_{o1}. Note Step 9.

B 4.5.2.1 Simple Bending about a Principal Axis-- Symmetrical Sections. (Cont'd)

4. Locate the neutral axis which would be some distance Δ from the principal axis toward the tension side. This may be difficult to determine, but trial and error type formulae are provided in Figure B4.5.2.1-2 for several symmetrical cross-sections. See Example B4.5.4.1 for typical procedure in determining Δ, ϵ_2, f_2, f_{o2}, k_1, and k_2. Note that k_1 and k_2 are with respect to the neutral axis.

5. Find F_{b1} and F_{b2} for ϵ_1 and ϵ_2, using the correct values of k_1 and k_2 which may or may not be equal.

6. Calculate I_1 and I_2, the moments of inertia of the elements with respect to the neutral axis of the entire cross-section.

7. Calculate M_{b_a} by

$$M_{b_a} = \frac{F_{b_1} I_1}{c_1} + \frac{F_{b_2} I_2}{c_2} \qquad (4.5.2.1\text{-}1)$$

8. For cases as in Step 2, F_b may be used with f_b, the calculated Mc/I stress, in determining the stress ratio for bending and the margin of safety for pure bending as follows:

$$R_b = \frac{f_b}{F_b} \qquad (4.5.2.1\text{-}2)$$

$$M.S. = \frac{1}{(S.F.)\, R_b} - 1 \qquad (4.5.2.1\text{-}3)$$

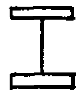
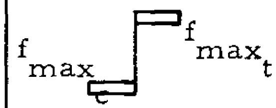
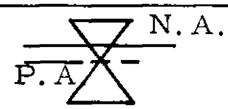
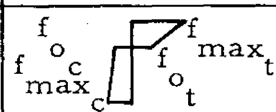
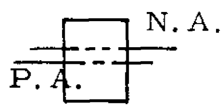
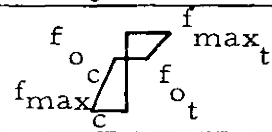
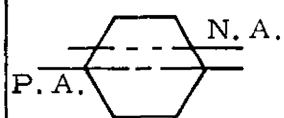
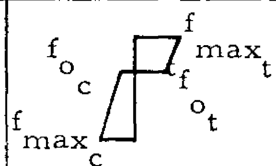
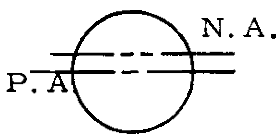
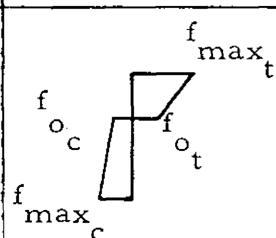
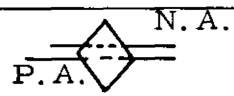
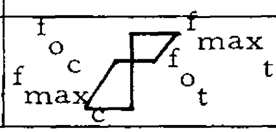
where:

S.F. is the appropriate (yield or ult) safety factor.

9. For cases as in Step 3, M_{b_a} <u>must</u> be used in determining the moment ratio for bending and the margin of safety for pure bending as follows:

$$R_b = \frac{M}{M_{b_a}} \qquad (4.5.2.1\text{-}4)$$

$$M.S. = \frac{1}{(S.F.)\, R_b} - 1 \qquad (4.5.2.1\text{-}5)$$

k	CROSS-SECTION	STRESS DISTRIBUTION	FORMULA (Correct Assumption of Δ Required to Satisfy the Equality of the Equation)
1.0			Calculation of Δ not required. Modulus of rupture is equal to the limiting value of stress.
1.33			$(1-\Delta)\left[\frac{2\Delta+1}{6} f_{o_t} + \frac{\Delta+2}{6} f_{max_t}\right] = \frac{3\Delta^2+1}{6} f_{o_c} + \frac{1}{3} f_{max_c}$
1.5			$(f_{max_t} + f_{o_t})(1-\Delta) = (f_{max_c} + f_{o_c})(1+\Delta)$
1.6			$(1-\Delta)\left[f_{o_t}\left(0.9623 - \frac{\Delta}{3}\right) + f_{max_t}\left(0.7698 - \frac{\Delta}{6}\right)\right] = f_{o_c}\left[\Delta\left(2.3094 - \frac{\Delta}{2}\right) + 0.9623\right] + 0.7698\, f_{max_c}$
1.7			$\left[f_{o_t} - \frac{\Delta}{1-\Delta}\left(f_{max_t} - f_{o_t}\right)\right]\left[\frac{\pi}{2} - \Delta\left(1-\Delta^2\right)^{1/2} - \sin^{-1}\Delta\right] + \frac{2}{3}\left[\frac{f_{max_t} - f_{o_t}}{1-\Delta}\right]\left(1-\Delta^2\right)^{3/2} = \left[f_{o_c} + \frac{\Delta}{1+\Delta}\left(f_{max_c} - f_{o_c}\right)\right] X \left[\frac{\pi}{2} + \Delta\left(1-\Delta^2\right)^{1/2} + \sin^{-1}\Delta\right] + \frac{2}{3}\left[\frac{f_{max_c} - f_{o_c}}{1+\Delta}\right]\left(1-\Delta^2\right)^{3/2}$
2.0			$f_t + 2f_{o_t} = \frac{1}{\left(1-\Delta^2\right)}\left[f_c + f_{o_c}\left(2 + 6\Delta - 3\Delta^2\right)\right]$

TRIAL AND ERROR FORMULAE FOR SHIFT OF NEUTRAL AXIS IN MATERIAL WHOSE TENSION AND COMPRESSION STRESS-STRAIN CURVES DIFFER SIGNIFICANTLY

FIGURE B4.5.2.1-2

B4.5.2.2 Simple Bending about a Principal Axis--Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

The procedure is as follows:

1. Locate the neutral axis which would be some distance Δ from the principal axis toward the tension side by a method similar to that outlined in Example B4.5.4.1 for symmetrical sections. This will require the derivation of an expression relating Δ, f_{max_t}, f_{o_t}, f_{max_c}, and f_{o_c} by use of the equilibrium of axial loads due to the bending stress distribution. This expression is likely to contain higher powers of Δ and the solution for Δ can best be obtained by trial and error using the stress-strain (or k = 1) and f_o curves. Refer to Example B4.5.4.2 for typical procedure.

2. Now, follow the identical procedure to that of Section B4.5.1.2 with the exception of using the neutral axis instead of the principal axis. Remember that F_b on the compression side is obtained from the compression Plastic Bending Curves.

B4.5.2.3 Complex Bending--Symmetrical Sections; also Unsymmetrical Sections with One Axis of Symmetry

Refer to Section B4.5.1.3 and follow the identical procedure except for Step 2 which is expressed as follows:

2. Follow the applicable procedure outlined in Section B4.5.2.1 and/or B4.5.2.2 to determine R_{b_x} and R_{b_y}.

B4.5.2.4 Complex Bending--Unsymmetrical Sections with No Axis of Symmetry

Refer to Section B4.5.1.4 and follow the identical procedure except for Step 3 which is expressed as follows:

3. Follow the procedure outlined in Section B4.5.2.2 to determine R_{b_x} and R_{b_y}

B4.5.2.5 Shear Flow for Simple Bending about a Principal Axis--Symmetrical Sections

Refer to Section B4.5.1.5 and follow the identical procedure with the following exception:

> If shear flow is being determined on the tension side, use the tension Plastic Bending Curves in the evaluation of λ for Equation (4.5.1.5-3). The compression side is considered similarly using the compression Plastic Bending Curves.

B4.5.2.6 Shear Flow for Simple Bending about a Principal Axis--Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

Refer to Section B4.5.1.6 and follow the identical procedure with the following exceptions:

> If shear flow is being determined on the tension side, use the tension Plastic Bending Curves in the evaluation of λ for Equations (4.5.1.6-1) or (4.5.1.6-2). The compression side is considered similarly using the compression Plastic Bending Curves.

B4.5.2.7 Shear Flow for Complex Bending--Any Cross-Section

Refer to Section B4.5.1.7 and follow the identical procedure except for Step 3 which is expressed as follows:

3. Follow the same procedure of Section B4.5.2.6 about both principal axes and obtain the shear stresses $f_{s_x'}$ and $f_{s_y'}$, <u>at a prescribed point</u>.

B4.5.3 The Effects of Transverse Stresses on Plastic Bending

The elastic and plastic portions of stress-strain curves for combined loading do not correspond to those of the uniaxial curves due to the effects of Poisson's ratio. The plastic bending curves depend on the magnitude and shape of the stesss-strain curve of a given material. Therefore, under combined loading the plastic bending curves may have to be modified if the uniaxial stress-strain curve is significantly affected.

B 4.5.3 The Effects of Transverse Stresses on Plastic Bending.

When modification of the plastic bending curves is necessary, the procedure is as follows:

1. Modify the uniaxial stress-strain curve by Section A3.7.0.

2. Determine and construct the modified plastic bending curves, F_b vs ϵ. Refer to Section B4.5.0 for the theoretical background; the f_o curve must first be plotted and then k curves constructed by the use of Equation (4.5.0-1).

B4.5.4 Example Problems

B4.5.4.1 A Diamond Cross-Section of AISI 301 Extra Hard (2% Elongation) Stainless Steel Sheet Is Subjected to Pure Bending in the Longitudinal Direction at Room Temperature. Find the Shift in Neutral Axis (Δ) from the Principal Axis When the Limiting Stress Is (a) F_{tu} and (b) F_{cy}. (F_{ty} is Is Not Used Since $F_{ty} > F_{cy}$.)

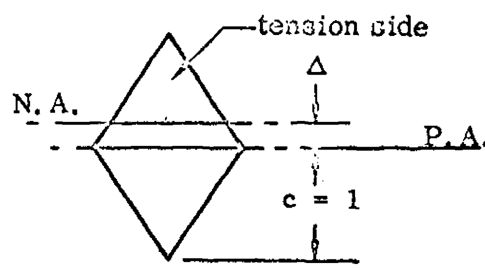
Reference the Minimum Plastic Bending Curves on page 133 and 134.

(a)

$F_{tu} = 200$ ksi

$\epsilon_t = .020$ in./in.

$f_{o_t} = 146$ ksi



From Figure B4.5.2.1-2 use the equation for determining Δ when k = 2 (for a diamond).

$$f_t + 2f_{o_t} = \frac{1}{(1 - \Delta^2)}[f_c + f_{o_c}(2 + 6\Delta - 3\Delta^2)]$$

By trial and error, equality is reached within 0.8% when Δ is assumed equal to 0.187c as shown below:

From Equation (4.5.1.2-3)

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \quad \text{or} \quad \epsilon_c = \frac{\epsilon_t c_c}{c_t}$$

$$\epsilon_c = \frac{0.020 \times 1.187c}{.813c} = 0.0292 \text{ in./in.}$$

B 4.5.4.1 Example Problem B 4.5.4.1 (Cont'd)

From the stress-strain (k=1) curve page 135 at ϵ_c

$$f_c = 149 \text{ ksi}$$

$$f_{o_c} = 107 \text{ ksi}$$

Therefore,

$$200 + 2 \times 146 \approx \frac{1}{1 - \overline{0.187}^2} [149 + 107(2 + 6 \times 0.187 - 3 \times \overline{0.187}^2]$$

$$492 \approx 488$$

(b) $f_{cy} = 97$ ksi

$$\epsilon_c = 0.00575 \text{ in./in.}$$

$$f_{o_c} = 27 \text{ ksi}$$

From Figure B4.5.2.1-2, use the equation for determining Δ when k = 2 (for a diamond).

$$f_t + 2 f_{o_t} = \frac{1}{(1 - \Delta^2)} [f_c + f_{o_c} (2 + 6\Delta - 3\Delta^2)]$$

By trial and error, equality is reached within 0.60% when Δ is assumed equal to 0.024c as shown below:

From Equation (4.5.1.2-3)

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \quad \text{or} \quad = \quad \epsilon_t = \frac{\epsilon_c c_t}{c_c}$$

$$\epsilon_t = \frac{0.00575 \times 0.976c}{1.024c} = 0.00548 \text{ in./in.}$$

From the stress-strain (k=1) curve page 132 at ϵ_t

$$f_t = 121 \text{ ksi}$$

$$f_{o_t} = 16.5 \text{ ksi}$$

B 4.5.4.1 Example Problem B 4.5.4.1 (Cont'd)

Therefore,

$$121 + 2 \times 16.5 \approx \frac{1}{\overline{1-0.024}^2}\left[97 + 27\left(2 + 6\times 0.024 - 3\times \overline{0.024}^2\right)\right]$$

$$154 \approx 154.93$$

B4.5.4.2 Calculate the Ultimate Allowable Bending Moment About the Principal Axis for the Built-Up Tee Section Shown. The Flange Is in Compression and the Crippling Stress Is 60 ksi. The Material Is AISI 301 ½ Hard Stainless Steel Sheet at Room Temperature with Bending in the Longitudinal Grain Direction.

Since the longitudinal tension and compression stress-strain curves are significantly different, the procedure of Section B4.5.2.2 will be followed.

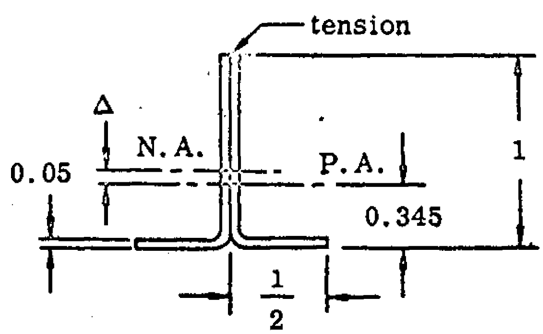
For ultimate design, crippling is the limiting stress rather than F_{tu}.

$F_{cc} = 60$ ksi

From page 114

$\epsilon_c = 0.0046$ in./in.

$f_{o_c} = 27$ ksi



A trial and error equation for shifting the neutral axis from the principal axis toward the tension side is determined by equating the load on the tension side to the load on the compression side. (See Section B4.5.2.2):

$$c_t\left(\frac{f_t + f_{o_t}}{2}\right) = c_c\left(\frac{F_{cc} + f_{o_c}}{2}\right) + \left(\frac{1}{2} - .05\right) F_{cc}$$

B 4.5.4.2 Example Problem B 4.5.4.2 (Cont'd)

Equality is reached within 0.70% when Δ is assumed equal to 0.065:

$$c_c = 0.345 + \Delta = 0.410 \text{ in.}$$

$$c_t = 1 - c_c = 0.590 \text{ in.}$$

Using Equation (4.5.1.2-3)

$$\epsilon_t = \frac{\epsilon_c c_t}{c_c} = \frac{0.0046 \times 0.590}{0.410} = 0.00662 \text{ in./in.}$$

From page 112 at ϵ_t

$$f_t = 113 \text{ ksi}$$

$$f_{o_t} = 38 \text{ ksi}$$

Substitution of these values into the above trial and error equation results in equality within 0.70%.

Section properties about the neutral axis:

$$I_t = \frac{2\,t\,c_t^3}{3} = \frac{2 \times 0.05 \times \overline{0.590}^3}{3} = 0.006845 \text{ in.}^4$$

$$Q_t = t\,c_t^2 = 0.05 \times \overline{0.590}^2 = 0.01740 \text{ in.}^3$$

$$k_t = \frac{Q_t c_t}{I_t} = \frac{0.01740 \times 0.590}{0.006845} = 1.5$$

(which checks with Figure B4.5.0-3 for a rectangle)

$$I_c = \frac{2\,t\,c_c^3}{3} + 2 \times 0.450t\left(c_c - \frac{t}{2}\right)^2 = \frac{2 \text{x} 0.05 \text{x} 0.410^3}{3}$$

$$+ 2 \text{x} 0.450 \text{x} 0.05 \text{x} \overline{0.385}^2 = 0.008967 \text{ in.}^4$$

B 4.5.4.2 Example Problem B 4.5.4.2 (Cont'd)

$$Q_c = t c_c^2 + 2 \times 0.450t \left(c_c - \frac{t}{2}\right) = 0.05 \times \overline{0.410}^2 + 2 \times 0.450$$

$$\times 0.05 \times 0.385 = 0.02572 \text{ in.}^3$$

$$k_c = \frac{Q_c c_c}{I_c} = \frac{0.02572 \times 0.410}{0.008967} = 1.17$$

From page 112 using ϵ_t and k_t

$$f_{b_t} = 132 \text{ ksi}$$

Let's check this value by numerically evaluating Equation (4.5.0-1).

$$F_{b_t} = f_t + (k - 1) f_{o_t} = 113 + (1.5-1)38 = 132 \text{ ksi (check)}$$

From page 114 using ϵ_c and k_c

$$F_{b_c} = 64.5 \text{ ksi}$$

Calculate the ultimate allowable moment by Equation (4.5.1.2-4)

$$M_{b_{ult}} = \frac{F_{b_t} I_t}{c_t} + \frac{F_{b_c} I_c}{c_c} = \frac{132 \times 0.006845}{0.590}$$

$$+ \frac{64.5 \times 0.008967}{0.410} = \underline{\underline{2.94 \text{ in. - kips}}} \text{. (answer)}$$

B4.5.4.3 Find the Shear Flow at the Neutral Axis of the Example Problem B4.5.4.2 If the Transverse Shear, (s), Is Equal to 5 kips. and the Bending Moment Is Equal to the Ultimate (2.94 in-kips.)

Shear flow at the neutral axis should be determined by using the section properties from each side of the neutral axis. The larger of the two shear flows should be (conservatively) used.

The procedure of Section B4.5.2.6 and consequently Section B4.5.1.6 will be used.

B4.5.4.3 Example Problem B4.5.4.3 (Cont'd)

Shear Flow From Tension Properties

The required section properties already known from example B4.5.4.2 are:

$I_t = 0.006845 \text{ in.}^4$ (tension side only)

$I_c = 0.008967 \text{ in.}^4$ (compression side only)

$I = I_t + I_c = 0.015812 \text{ in.}^4$

$c_t = 0.590 \text{ in.}$

$Q_t = 0.01740 \text{ in.}^3$

$k_t = 1.5$ (rectangle)

$\epsilon_t = 0.00662 \text{ in./in.}$

From the stress-strain (k= 1) curve on page 112 at ϵ_t

$$\left(\frac{df}{d\epsilon}\right)_1 = 7 \text{ ksi}/.001$$

$$\left(\frac{df_o}{d\epsilon}\right)_1 = 8 \text{ ksi}/.001$$

Using Equation (4.5.1.6-1)

$$\lambda_1 = \left(\frac{df_o}{df}\right)_1 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_1$$

$$\lambda_1 = \frac{8}{7} = 1.14$$

Using Equation (4.5.1.6-4)

$$\beta_a = \frac{1 + \lambda_1\left(\frac{c_1}{\bar{y}_a} - 1\right)}{1 + \lambda_1 (k_1 - 1)}$$

$$\bar{y}_a = \frac{c_t}{2} = 0.295 \text{ in.}$$

B 4.5.4.3 Example Problem B 4.5.4.3 (Cont'd)

$$\beta_a = \frac{1 + 1.14\left(\frac{0.590}{0.295} - 1\right)}{1 + 1.14\,(1.5 - 1)} = 1.36$$

Shear flow at the NA from Equation (4.5.1.5-2)

$$q_a = \frac{\beta_a\, SQ_a}{I}$$

$$q_a = 1.36\ \frac{5 \times 0.01740}{0.015812} = 7.48 \text{ kips/in.}$$

[Note that the conventional $\frac{SQ}{I}$ formula is 36% unconservative here.]

Shear Flow From Compression Properties

The required section properties already known from example B4.5.4.2 are:

$I = 0.015812 \text{ in.}^4$ (Reference Page 82)

$c_c = 0.410$ in.

$Q_c = 0.02572 \text{ in.}^3$

$k_c = 1.17$

$\epsilon_c = 0.0046$ in./in.

From the stress-strain (k = 1) curve on page 114 at ϵ_c

$$\left(\frac{df}{d\epsilon}\right)_2 = 4.6 \text{ ksi/.001}$$

$$\left(\frac{df_o}{d\epsilon}\right)_2 = 5.0 \text{ ksi/.001}$$

B 4.5.4.3 Example Problem B 4.5.4.3 (Cont'd)

Using Equation (4.5.1.6-2)

$$\lambda_2 = \left(\frac{df_o}{df}\right)_2 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_2$$

$$\lambda_2 = \frac{5.0}{4.6} = 1.09$$

Using Equation (4.5.1.6-7)

$$\beta_b = \frac{1 + \lambda_2\left(\frac{c_2}{\bar{y}_b} - 1\right)}{1 + \lambda_2 (k_2 - 1)}$$

$$\bar{y}_b = \frac{Q_b}{A_b} = \frac{Q_c}{A_c}$$

$$A_c = 2\,[0.05\,(0.410 + 0.450)] = 0.086 \text{ in.}^2$$

$$\bar{y}_b = \frac{0.02572}{0.086} = 0.299 \text{ in.}$$

$$\beta_b = \frac{1 + 1.09\left(\frac{0.410}{0.299} - 1\right)}{1 + 1.09(1.17 - 1)} = 1.18$$

Shear flow at the NA from Equation (4.5.1.6-6).

$$q_b = \beta_b \frac{SQ_b}{I}$$

$$q_b = 1.18 \frac{5 \times 0.02572}{0.015812} = 9.6 \text{ kips/in.}$$

[Note that the conventional $\frac{SQ}{I}$ formula is 18% unconservative here.]

The larger shear flow should be used which is

$q = 9.6$ kips/in.

B4.5.5 Index for Bending Modulus of Rupture Curves for Symmetrical Sections

These curves provide yield and ultimate modulus of rupture values for symmetrical sections only. For materials with significantly different tension and compression stress-strain curves, the necessary corrections for shifting of the neutral axis are already included. In the case of work hardened stainless steels in longitudinal bending with all fibers in tension (as in pressurized cylinders), the transverse Modulus of Rupture Curves are applicable.

It is recommended that MIL-HDBK-5 or other official sources be used for allowable material properties. Where these values correspond directly to the values called out on the graphs of this section, the modulus of rupture values are applicable as shown.

Where material allowables vary with thickness, cross-sectional area, etc., only one or two Modulus of Rupture Curves are presented. Therefore, for material properties slightly higher or lower than those used in the given curve, the modulus values may be ratioed up or down (provided the % elongations are practically the same).

B4.5.5.1 Stainless Steels - Minimum Properties

*(RT) - Room Temperature.

** Not presently available

B4.5.5.1 Stainless Steels - Minimum Properties

B4.5.5.2 Low Carbon and Alloy Steels * - Minimum Properties

B4.5.5.3 Heat Resistant Alloys - Minimum Properties

B4.5.5.4 Titanium - Minimum Properties

*Alloy Steels include AISI 4130, 4140, 4340, 8630, 8735, 8740, and 9840.

B4.5.6 Index for Plastic Bending Curves

These curves represent modulus of rupture values relative to the stress-strain curve for a section on either side of the neutral axis. These curves are particularly useful for unsymmetrical sections and when the allowable stress is other than yield or ultimate for symmetrical or unsymmetrical sections.

For materials with significantly different tension and compression stress-strain curves, Plastic Bending Curves for both are presented. When the difference is slight, the Plastic Bending Curves for the lower stress-strain curve only are presented.

It is recommended that MIL-HDBK-5 or other official sources be used for allowable material properties. Where these values correspond directly to the values called out on the graphs of this section, the modulus of rupture values are applicable as shown.

Where material allowables vary with thickness, cross-sectional area, etc., only one or two Plastic Bending Curves are presented. Therefore, for material properties slightly higher or lower than those used in the given curve, the modulus values may be ratioed up or down (provided the % elongations are practically the same).

Stress directions called out in the headings of the Plastic Bending Curves apply only to stress caused by pure bending. The bending modulus for compression is higher in the transverse grain direction (in fact, slightly higher than the tension modulus) than in the longitudinal grain direction, for the AISI 301 stainless steels. Separate curves are therefore presented for AISI stainless steels in the longitudinal compression.

B4.5.6.1 Stainless Steels-Minimum Properties

B4.5.6.2 Low Carbon and Alloy Steels**-Minimum Properties

* (RT) - Room Temperature.

** Alloy Steels include AISI 4130, 4140, 4340, 8630, 8735, 8740 and 9840.

B4.5.6.2 Low Carbon and Alloy Steels - Minimum Properties (Cont'd)

B4.5.6.3 Corrosion Resistant Metals-Minimum Properties

B4.5.6.4 Titanium-Minimum Properties

B4.5.6.5 Aluminum-Minimum Properties

B4.5.6.5 Aluminum - Minimum Properties (Cont'd)

B4.5.6.6 Magnesium-Minimum Properties