1.2 METHODS FOR MEASURING PERFORMANCE OF A MATERIAL

1.2.1 Normal Probability Curve

1.2.1.1 Properties

The normal, or Gaussian, curve is a two-parameter curve (Fig. H1-3). It is defined by the mean which locates the curve and the deviation which defines the spread of the curve. The area under the curve is always equal to one. The relation of the mean (μ), the standard deviation (σ), and the significance level (α) is shown in Fig. H1-4.

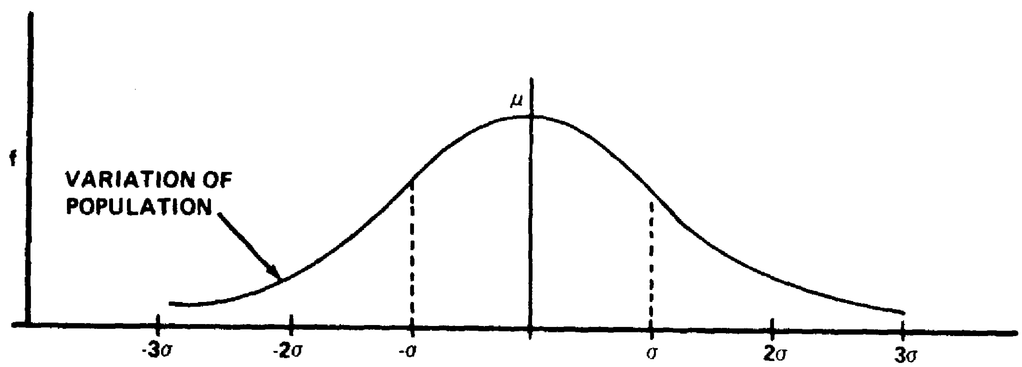

FIGURE H1-3. NORMAL (GAUSSIAN) DISTRIBUTION CURVE.

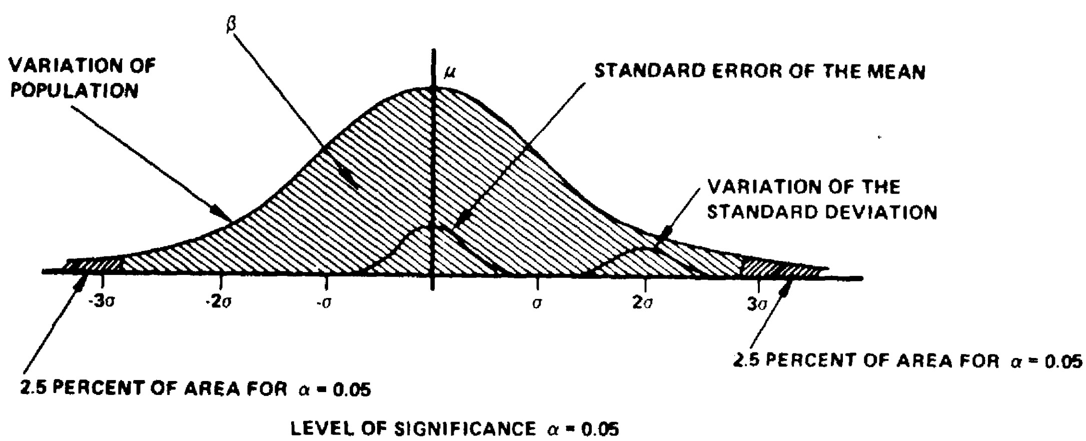

FIGURE H1-4. PARAMETERS AND PERFORMANCE GUIDES FOR NORMAL DISTRIBUTION.

1.2.1.2 Estimate of Average Performance

The most common and ordinarily the best single estimate of the population mean, m , is simply the arithmetic mean of the measurements.

$$m \approx \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad . \qquad (1)$$

1.2.1.3 Example Problem 1

Determine the mean value of the ultimate strength of product "A" having the test results in Table H1-1 [3].

Table H1-1. Ultimate Strength of Product "A"

Test Specimen	Ultimate Strength x_i (lb)
1	578
2	572
3	570
4	568
5	572
6	570
7	570
8	572
9	596
10	584
n = 10	$\sum x_i = 5752$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{10} (5752) = 575.2 \text{ lb} = \text{mean value}$$

1.2.1.4 Estimate of Standard Deviation

The estimate of the standard deviation is usually taken as the square root of the best unbiased estimate of variance [4].

$$\sigma \approx s = \sqrt{\frac{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2}{n(n - 1)}} \qquad (2)$$

1.2.1.5 Confidence Interval Estimate

When we take a sample from a lot or a population, the sample average will seldom be exactly the same as the lot or population average. We do hope that it is fairly close, and we would like to state an interval which we are confident will bracket the lot mean. If our intervals included the true average 95 percent of the time, we would be operating at a 95 percent confidence level, and our intervals would be called 95 percent confidence intervals. In general, if in the long run we expect 100 $(1 - \alpha)$ percent of our intervals to contain the true value, we are operating at the 100 $(1 - \alpha)$ percent confidence level. Confidence levels γ commonly used are 99 percent and 95 percent which correspond to $\alpha = 0.01$ and $\alpha = 0.05$.

1.2.1.6 Example Problem 2

Using the data in Table H1-1, what is the two-sided 100 $(1 - \alpha)$ percent confidence interval for the true mean m of the total population [4] ?

Choose desired confidence level, $1 - \alpha$	Let $1 - \alpha = 0.95$ $\alpha = 0.05$
Compute $\bar{x}$	$\bar{x} = 575.2$ lb
Compute s from Eq. (2)	$s = 8.24$
Look up $t = t_{1 - \frac{\alpha}{2}}$ for n - 1 degrees of freedom in Table H1-2	$t = t_{0.975}$ for 9 deg of freedom $= 2.262$
Compute $x_u = \bar{x} + t \frac{s}{\sqrt{n}}$	$x_u = 575.2 + \frac{2.262\ (8.24)}{\sqrt{10}}$ $= 581.1$ lb
Compute $x_L = \bar{x} - t \frac{s}{\sqrt{n}}$	$x_L = 575.2 - \frac{2.262\ (8.24)}{\sqrt{10}}$ $= 569.3$ lb

Conclude: The interval x_L to x_u is a $100\ (1 - \alpha)$ percent confidence interval for the population mean; i.e., we may assert with 95 percent confidence that the population mean is between 569.3 lb and 581.1 lb.

1.2.1.7 Example Problem 3

Using the data in Table H1-1, what is a one-sided $100\ (1 - \alpha)$ percent confidence interval for the true population mean [4] ?

Choose desired confidence level, $1 - \alpha$	Let $1 - \alpha = 0.99$ $\alpha = 0.01$
Compute $\bar{x}$	$\bar{x} = 575.7$ lb
s	$s = 8.24$

Table III-2. Percentiles of the "t" Distribution

df	$t_{0.60}$	$t_{0.70}$	$t_{0.80}$	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.690	2.326	2.576

Look up $t = t_{1-\alpha}$ for $n - 1$ deg of freedom in Table H1-2	$t = t_{0.99}$ for 9 deg of freedom $= 2.821$
Compute $x_L' = \bar{x} - t \frac{s}{\sqrt{n}}$ $\left(\text{or compute } x_u' = x + t \frac{s}{\sqrt{n}}\right)$	$x_L' = 575.7 - \frac{2.821\ (8.24)}{\sqrt{10}}$ $= 568.4$ lb

Conclude: We are $100\ (1 - \alpha)$ percent confident that the population mean is greater than x_L'; i.e., we may assert with 99 percent confidence that the population mean is greater than 568.4 lb.

1.2.1.8 Estimating Variability Example Problem

We have estimated the standard deviation; and, in a manner similar to determining the confidence interval for the mean, we may determine a confidence interval for the deviation which is termed the variability. Using the data in Table H1-1, determine an interval which brackets the true value of the standard deviation [4].

Choose the desired confidence level, $1 - \alpha$	Let $1 - \alpha = 0.95$ $\alpha = 0.05$
Compute s	$s = 8.24$
Look up B_u and B_L for $n - 1$ deg of freedom in Table H1-3	for 9 deg of freedom $B_L = 0.6657$ $B_u = 1.746$

Table H1-3. Factors for Computing Two-sided Confidence Limits for σ

Degrees of Freedom df	α = 0.05		α = 0.01		α = 0.001	
	B_U	B_L	B_U	B_L	B_U	B_L
1	17.79	0.3576	86.31	0.2969	844.4	0.2480
2	4.859	0.4581	10.70	0.3879	33.29	0.3291
3	3.183	0.5178	5.449	0.4453	11.65	0.3824
4	2.567	0.5590	3.892	0.4865	6.938	0.4218
5	2.248	0.5899	3.175	0.5182	5.085	0.4529
6	2.052	0.6143	2.764	0.5437	4.128	0.4784
7	1.918	0.6344	2.498	0.5650	3.551	0.5000
8	1.820	0.6513	2.311	0.5830	3.167	0.5186
9	1.746	0.6657	2.173	0.5987	2.894	0.5348
10	1.686	0.6784	2.065	0.6125	2.689	0.5492
11	1.638	0.6896	1.980	0.6248	2.530	0.5621
12	1.598	0.6995	1.909	0.6358	2.402	0.5738
13	1.564	0.7084	1.851	0.6458	2.298	0.5845
14	1.534	0.7166	1.801	0.6549	2.210	0.5942
15	1.509	0.7240	1.758	0.6632	2.136	0.6032
16	1.486	0.7308	1.721	0.6719	2.073	0.611[illegible]
17	1.466	0.7372	1.688	0.6781	2.017	0.6183
18	1.448	0.7430	1.658	0.6848	1.968	0.6266
19	1.432	0.7484	1.632	0.6909	1.925	0.6333
20	1.417	0.7535	1.609	0.6968	1.886	0.6397
21	1.404	0.7582	1.587	0.7022	1.851	0.6457
22	1.391	0.7627	1.568	0.7074	1.820	0.6514
23	1.380	0.7669	1.550	0.7122	1.791	0.6568
24	1.370	0.7709	1.533	0.7169	1.765	0.6619
25	1.360	0.7747	1.518	0.7212	1.741	0.6668
26	1.351	0.7783	1.504	0.7253	1.719	0.6713
27	1.343	0.7817	1.491	0.7293	1.698	0.6758
28	1.335	0.7849	1.479	0.7331	1.679	0.6800
29	1.327	0.7880	1.467	0.7367	1.661	0.6841
30	1.321	0.7909	1.457	0.7401	1.645	0.6880
31	1.314	0.7937	1.447	0.7434	1.629	0.6917
32	1.308	0.7964	1.437	0.7467	1.615	0.6953
33	1.302	0.7990	1.428	0.7497	1.601	0.6987
34	1.296	0.8015	1.420	0.7526	1.588	0.7020
35	1.291	0.8039	1.412	0.7554	1.576	0.7052
36	1.286	0.8062	1.404	0.7582	1.564	0.7083
37	1.281	0.8085	1.397	0.7608	1.553	0.7113
38	1.277	0.8106	1.390	0.7633	1.543	0.7141
39	1.272	0.8126	1.383	0.7658	1.533	0.7169
40	1.268	0.8146	1.377	0.7681	1.523	0.7197
41	1.264	0.8166	1.371	0.7705	1.515	0.7223
42	1.260	0.8184	1.365	0.7727	1.506	0.7248
43	1.257	0.8202	1.360	0.7748	1.498	0.7273
44	1.253	0.8220	1.355	0.7769	1.490	0.7297
45	1.249	0.8237	1.349	0.7789	1.482	0.7320
46	1.246	0.8253	1.345	0.7809	1.475	0.7342
47	1.243	0.8269	1.340	0.7828	1.468	0.7364
48	1.240	0.8285	1.335	0.7847	1.462	0.7386
49	1.237	0.8300	1.331	0.7864	1.455	0.7407
50	1.234	0.8314	1.327	0.7882	1.449	0.7427

Table H1-3 (Continued)

Degrees of Freedom df	α = 0.05		α = 0.01		α = 0.001	
	B_U	B_L	B_U	B_L	B_U	B_L
51	1.232	0.8329	1.323	0.7899	1.443	0.7446
52	1.229	0.8343	1.319	0.7916	1.437	0.7466
53	1.226	0.8356	1.315	0.7932	1.432	0.7485
54	1.224	0.8370	1.311	0.7949	1.426	0.7503
55	1.221	0.8383	1.308	0.7964	1.421	0.7521
56	1.219	0.8395	1.304	0.7979	1.416	0.7539
57	1.217	0.8408	1.301	0.7994	1.411	0.7556
58	1.214	0.8420	1.298	0.8008	1.406	0.7573
59	1.212	0.8431	1.295	0.8022	1.402	0.7589
60	1.210	0.8443	1.292	0.8036	1.397	0.7605
61	1.208	0.8454	1.289	0.8050	1.393	0.7621
62	1.206	0.8465	1.286	0.8063	1.389	0.7636
63	1.204	0.8475	1.283	0.8076	1.385	0.7651
64	1.202	0.8486	1.280	0.8088	1.381	0.7666
65	1.200	0.8496	1.277	0.8101	1.377	0.7680
66	1.199	0.8506	1.275	0.8113	1.374	0.7694
67	1.197	0.8516	1.272	0.8125	1.370	0.7708
68	1.195	0.8525	1.270	0.8137	1.366	0.7722
69	1.194	0.8535	1.268	0.8148	1.363	0.7735
70	1.192	0.8544	1.265	0.8159	1.360	0.7749
71	1.190	0.8553	1.263	0.8170	1.356	0.7761
72	1.189	0.8562	1.261	0.8181	1.353	0.7774
73	1.187	0.8571	1.259	0.8191	1.350	0.7787
74	1.186	0.8580	1.257	0.8202	1.347	0.7799
75	1.184	0.8588	1.255	0.8212	1.344	0.7811
76	1.183	0.8596	1.253	0.8222	1.341	0.7822
77	1.182	0.8604	1.251	0.8232	1.338	0.7834
78	1.181	0.8612	1.249	0.8242	1.336	0.7845
79	1.179	0.8620	1.247	0.8252	1.333	0.7856
80	1.178	0.8627	1.245	0.8261	1.330	0.7868
81	1.176	0.8635	1.243	0.8270	1.328	0.7878
82	1.176	0.8642	1.241	0.8279	1.325	0.7889
83	1.174	0.8650	1.239	0.8288	1.323	0.7899
84	1.174	0.8657	1.238	0.8297	1.320	0.7909
85	1.172	0.8664	1.236	0.8305	1.318	0.7920
86	1.171	0.8671	1.235	0.8314	1.316	0.7930
87	1.170	0.8678	1.233	0.8322	1.313	0.7939
88	1.168	0.8684	1.231	0.8331	1.311	0.7949
89	1.167	0.8691	1.230	0.8338	1.309	0.7959
90	1.166	0.8697	1.228	0.8346	1.307	0.7968
91	1.165	0.8704	1.227	0.8354	1.305	0.7977
92	1.164	0.8710	1.225	0.8362	1.303	0.7987
93	1.163	0.8716	1.224	0.8370	1.301	0.7996
94	1.162	0.8722	1.222	0.8377	1.298	0.8004
95	1.161	0.8729	1.221	0.8385	1.297	0.8013
96	1.160	0.8734	1.219	0.8392	1.295	0.8022
97	1.159	0.8741	1.218	0.8399	1.293	0.8031
98	1.158	0.8746	1.217	0.8406	1.291	0.8039
99	1.158	0.8752	1.216	0.8413	1.289	0.8047
100	1.157	0.8757	1.214	0.8420	1.288	0.8055

Compute $s_L = B_L s$	$s_L = (8.24)\ (0.6657)$
	$= 5.48$
$s_u = B_u s$	$s_u = (8.24)\ (1.746)$
	$= 14.38$

Conclude: The interval from s_L to s_u is a two-sided $100\ (1 - \alpha)$ percent confidence interval estimate for σ; i.e., we may assert with 95 percent confidence that σ is between 5.48 and 14.38.

1.2.1.9 Number of Measurements Required

In planning experiments, we may need to know how many measurements to take in order to determine a parameter of some distribution with prescribed accuracy. If an estimate s or σ is available or if we are willing to assume a σ; we may ascertain the required sample size n for determining the mean [4].

Choose d, the allowable margin of error, and α, the risk that our estimate of m will be off by d or more	Let $d = 0.2$ $\alpha = 0.05$
Look up $t = t_{1 - \frac{\alpha}{2}}$ for df deg of freedom in Table H1-2	$t = t_{0.975}$ for 9 deg of freedom $= 2.262$

Compute $n = \frac{t^2 s^2}{d^2}$	$n = \frac{(2.262)^2 (8.24)^2}{(0.2)^2}$ 8700

Conclude: We may conclude that if we now compute the mean $\bar{x}$ of a random sample of size $n = 8700$, we may have 95 percent confidence that the interval $\bar{x} - 0.2$ to $\bar{x} + 0.2$ will include the lot mean.

A similar procedure may be used when computing the standard deviation. As an example, how large a sample would be required to estimate the standard deviation within 20 percent of its true value, with confidence coefficient equal to 0.95 [4] ?

Specify P , the allowable percentage deviation of the estimated standard deviation from its true value	Let $P = 20$ percent
Choose γ , the confidence coefficient	Let $\gamma = 0.95$
In Figure H1-5, find P on the horizontal scale, and use the curve for the appropriate γ . Read on the vertical scale the required degree of freedom	For $\gamma = 0.95$, $P = 20$ percent $df = 46$
$n = df + 1$	$n = 46 + 1 = 47$

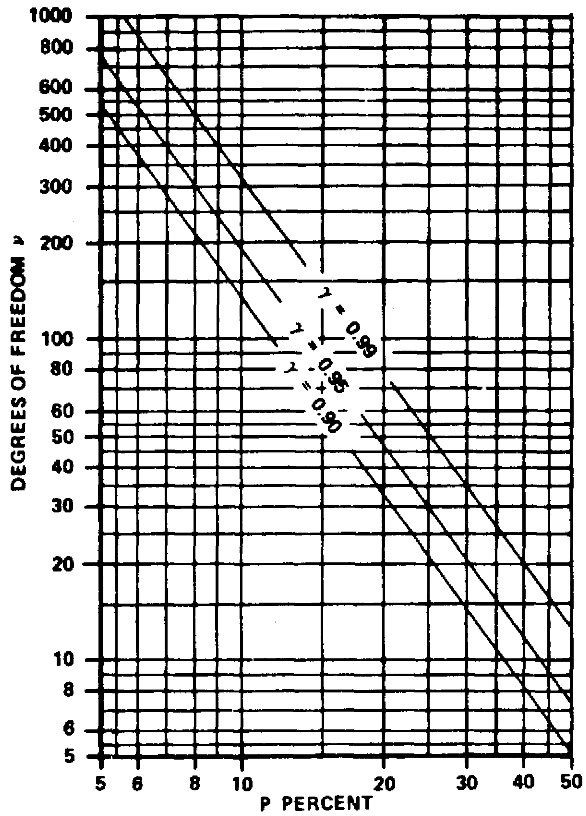

FIGURE H1-5. NUMBER OF DEGREES OF FREEDOM REQUIRED TO ESTIMATE THE STANDARD DEVIATION WITHIN P PERCENT OF ITS TRUE VALUE WITH CONFIDENCE COEFFICIENT γ.

1.2.1.10 Tolerance Limits

Sometimes we are more interested in the approximate range of values in a lot or population than we are its average value. Statistical tolerance limits furnish limits between, above, or below which we confidently expect to find a prescribed proportion of individual items of a population. Thus, we might be able to give a value A above which at least a proportion P of the population will lie (one-sided limit). In this case, $x_L = \bar{x} - Ks$ will be the one-sided lower limit. The appropriate values for K are given in Table H1-4 [4]. As an example, consider the data in example problem 1 and find a

Table H1-4. Factors for One-sided Tolerance Limits for Normal Distributions

Factors K such that the probability is γ that at least a proportion P of the distribution will be less than $\bar{x}$ + Ks (or greater than $\bar{x}$ - Ks), where $\bar{x}$ and s are estimates of the mean and the standard deviation computed from a sample size of n .

	γ = 0.75					γ = 0.90				
n \ P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
3	1.464	2.501	3.152	4.396	5.805	2.602	4.258	5.310	7.340	9.651
4	1.256	2.134	2.680	3.726	4.910	1.972	3.187	3.957	5.437	7.128
5	1.152	1.961	2.463	3.421	4.507	1.698	2.742	3.400	4.666	6.112
6	1.087	1.860	2.336	3.243	4.273	1.540	2.494	3.091	4.242	5.556
7	1.043	1.791	2.250	3.126	4.118	1.435	2.333	2.894	3.972	5.201
8	1.010	1.740	2.190	3.042	4.008	1.360	2.219	2.755	3.783	4.955
9	0.984	1.702	2.141	2.977	3.924	1.302	2.133	2.649	3.641	4.772
10	0.964	1.671	2.103	2.927	3.858	1.257	2.065	2.568	3.532	4.629
11	0.947	1.646	2.073	2.885	3.804	1.219	2.012	2.503	3.444	4.515
12	0.933	1.624	2.048	2.851	3.760	1.188	1.966	2.448	3.371	4.420
13	0.919	1.606	2.026	2.822	3.722	1.162	1.928	2.403	3.310	4.341
14	0.909	1.591	2.007	2.796	3.690	1.139	1.895	2.363	3.257	4.274
15	0.899	1.577	1.991	2.776	3.661	1.119	1.866	2.329	3.212	4.215
16	0.891	1.566	1.977	2.756	3.637	1.101	1.842	2.299	3.172	4.164
17	0.883	1.554	1.964	2.739	3.615	1.085	1.820	2.272	3.136	4.118
18	0.876	1.544	1.951	2.723	3.595	1.071	1.800	2.249	3.106	4.078
19	0.870	1.536	1.942	2.710	3.577	1.058	1.781	2.228	3.078	4.041
20	0.865	1.528	1.933	2.697	3.561	1.046	1.765	2.208	3.052	4.009
21	0.859	1.520	1.923	2.686	3.545	1.035	1.750	2.190	3.028	3.979
22	0.854	1.514	1.916	2.675	3.532	1.025	1.736	2.174	3.007	3.952
23	0.849	1.508	1.907	2.665	3.520	1.016	1.724	2.159	2.987	3.927
24	0.845	1.502	1.901	2.656	3.509	1.007	1.712	2.145	2.969	3.904
25	0.842	1.496	1.895	2.647	3.497	0.999	1.702	2.132	2.952	3.882
30	0.825	1.475	1.869	2.613	3.454	0.966	1.657	2.080	2.884	3.794
35	0.812	1.458	1.849	2.588	3.421	0.942	1.623	2.041	2.833	3.730
40	0.803	1.445	1.834	2.568	3.395	0.923	1.598	2.010	2.793	3.679
45	0.795	1.435	1.821	2.552	3.375	0.908	1.577	1.986	2.762	3.638
50	0.788	1.426	1.811	2.538	3.358	0.894	1.560	1.965	2.735	3.604

Table H1-4. (Continued)

	γ = 0.95					γ = 0.99				
n \ P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
3	3.804	6.158	7.655	10.552	13.857	—	—	—	—	—
4	2.619	4.163	5.145	7.042	9.215	—	—	—	—	—
5	2.149	3.407	4.202	5.741	7.501	—	—	—	—	—
6	1.895	3.006	3.707	5.062	6.612	2.849	4.408	5.409	7.334	9.540
7	1.732	2.755	3.399	4.641	6.061	2.490	3.856	4.730	6.411	8.348
8	1.617	2.582	3.188	4.353	5.686	2.252	3.496	4.287	5.811	7.566
9	1.532	2.454	3.031	4.143	5.414	2.085	3.242	3.971	5.389	7.014
10	1.465	2.355	2.911	3.981	5.203	1.954	3.048	3.739	5.075	6.603
11	1.411	2.275	2.815	3.852	5.036	1.854	2.897	3.557	4.828	6.284
12	1.366	2.210	2.736	3.747	4.900	1.771	2.773	3.410	4.633	6.032
13	1.329	2.155	2.670	3.659	4.787	1.702	2.677	3.290	4.472	5.826
14	1.296	2.108	2.614	3.585	4.690	1.645	2.592	3.189	4.336	5.651
15	1.268	2.068	2.566	3.520	4.607	1.596	2.521	3.102	4.224	5.507
16	1.242	2.032	2.523	3.463	4.534	1.553	2.458	3.028	4.124	5.374
17	1.220	2.001	2.486	3.415	4.471	1.514	2.405	2.962	4.038	5.268
18	1.200	1.974	2.453	3.370	4.415	1.481	2.357	2.906	3.961	5.167
19	1.183	1.949	2.423	3.331	4.364	1.450	2.315	2.855	3.893	5.078
20	1.167	1.926	2.396	3.295	4.319	1.424	2.275	2.807	3.832	5.003
21	1.152	1.905	2.371	3.262	4.276	1.397	2.241	2.768	3.776	4.932
22	1.138	1.887	2.350	3.233	4.238	1.376	2.208	2.729	3.727	4.866
23	1.126	1.869	2.329	3.206	4.204	1.355	2.179	2.693	3.680	4.806
24	1.114	1.853	2.309	3.181	4.171	1.336	2.154	2.663	3.638	4.755
25	1.103	1.838	2.292	3.158	4.143	1.319	2.129	2.632	3.601	4.706
30	1.059	1.778	2.220	3.064	4.022	1.249	2.029	2.516	3.446	4.508
35	1.025	1.732	2.166	2.994	3.934	1.195	1.957	2.431	3.334	4.364
40	0.999	1.697	2.126	2.941	3.866	1.154	1.902	2.365	3.250	4.255
45	0.978	1.669	2.092	2.897	3.811	1.122	1.857	2.313	3.181	4.168
50	0.961	1.646	2.065	2.863	3.766	1.096	1.821	2.296	3.124	4.096

single value above which we may predict with 90 percent confidence that 99 percent of the population will lie.

Choose P the proportion, and γ the confidence coefficient	Let $P = 0.99$ $\gamma = 0.90$
Compute $\bar{x}$	$\bar{x} = 575.7$ lb
s	$s = 8.24$
Look up K in Table H1-4 for appropriate n, γ, and P	$n = 10$, $P = 0.99$, $\gamma = 0.90$ $K = 3.532$
Compute $x_L = \bar{x} - Ks$	$x_L = 546.6$ lb Thus we are 90 percent confident that 99 percent of the material ultimate strengths for product "A" will be above 546.6 lb.

This same procedure is used in Ref. [6] to determine the "A" and "B" values for the primary strength properties (F_{tu}, F_{ty}, F_{cy}, F_{su}, F_{bru}, and F_{bry}).

1.2.2 Log-Normal Probability Curve

1.2.2.1 Properties

A log-normal distribution is the frequency distribution curve resulting from the use of the logarithm of a variable rather than the variable itself. The log-normal distribution curve is bell shaped like the normal probability curve and has the properties of a normal distribution.

1.2.2.2 Estimate of Average Performance

Similar to the normally distributed variables, the best single estimate of the population mean, m , is simply the arithmetic mean of the measurements [3].

$$\log m \approx \log \bar{x} = \frac{1}{n} \sum_{i=1}^{n} \log x_i \qquad (3)$$

1.2.2.3 Example Problem 4

Using the fatigue lives for a constant stress level (50 ksi) given in Table H1-5, estimate the mean of the parent universe [3].

Table H1-5. Fatigue Life of Product "B"

Test Specimen	N_i (Cycles)	Log N_i
1	13000	4.1139
2	13100	4.1173
3	24000	4.3802
4	28000	4.4472
5	40000	4.6021
n = 5		$\sum \log N_i$ = 21.6607

$$\overline{\log N} = \frac{1}{n} \sum \log N_i = \frac{1}{5} (21.6607) = 4.3321 = \text{Mean log value}$$

$$\bar{N} = \text{antilog } 4.3321 = 21485 \text{ cycles}$$

1.2.2.4 Estimate of Standard Deviation

Again, similar to normally distributed variables, the best unbiased estimate of the standard deviation is the square root of the estimate of variance [3].

$$\sigma \approx s \approx \sqrt{\frac{n \sum_{i=1}^{n} \left(\log x_i\right)^2 - \left(\sum_{i=1}^{n} \log x_i\right)^2}{n\,(n-1)}} \quad (4)$$

1.2.2.5 Interval Estimates

Confidence intervals may be determined for log-normal distributions. The range of a percentage of data points may be computed for the sample mean coinciding with the population mean, which occurs with a confidence of 50 percent. The range is computed as

$$\log x = \overline{\log x} \pm k_P\, s \quad , \quad (5)$$

where k_P is defined in Fig. H1-6. A lower limit (one-sided confidence interval) may be computed as

$$\log x_L = \overline{\log x} - k_{P\gamma} s \quad , \quad (6)$$

where $k_{P\gamma}$ is defined in Figs. H1-7 through H1-9 as a function of the sample size, the percentage of data points occurring above the lower limit, and the confidence value.

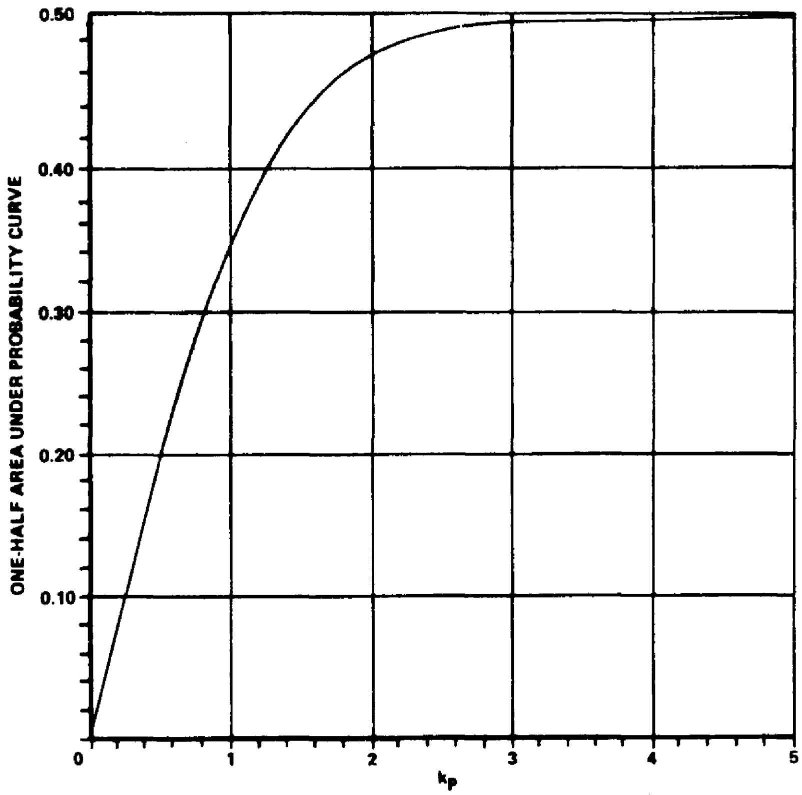

FIGURE H1-6. FACTOR FOR DETERMINING DATA POINT RANGE.

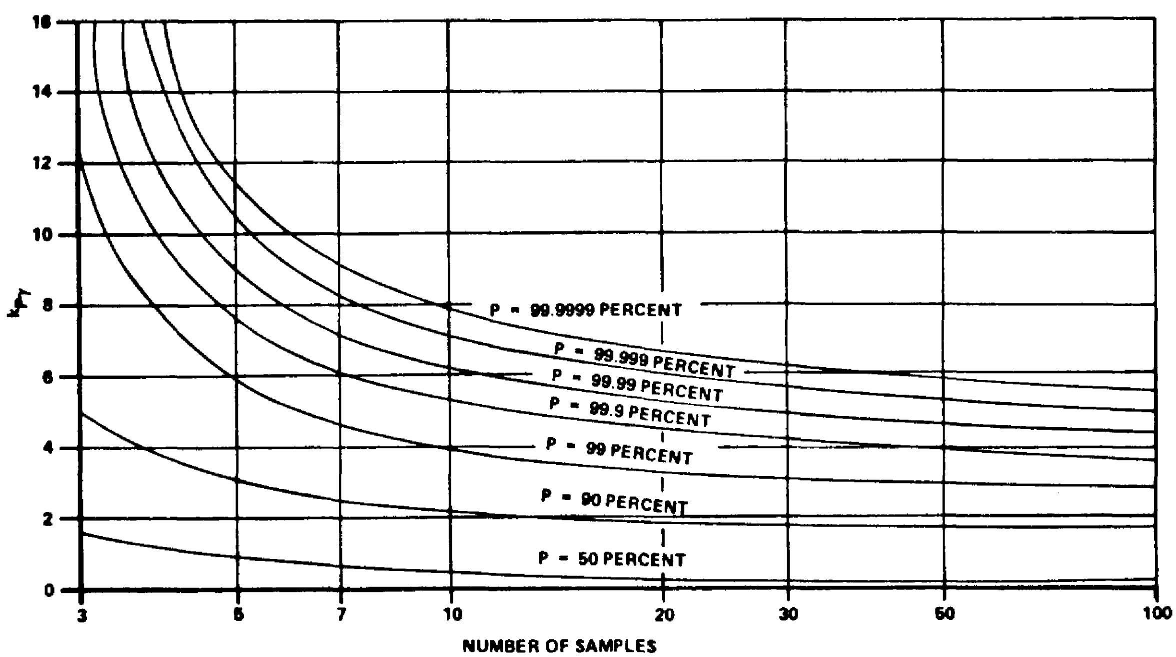

FIGURE H1-7. PROBABILITY FACTOR $k_{P\gamma}$ FOR $\gamma = 0.95$.

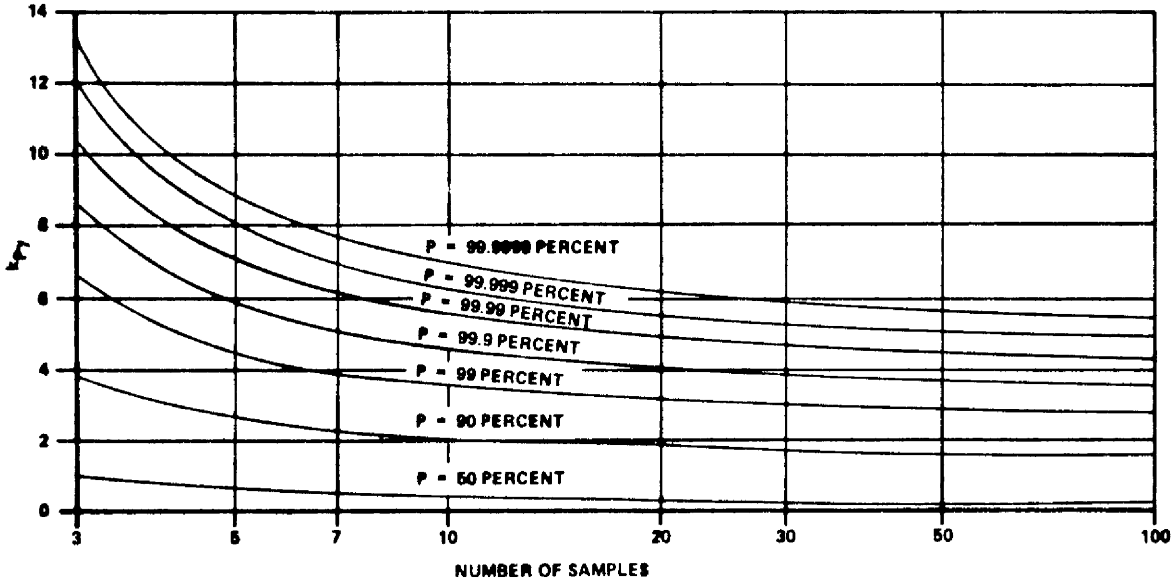

FIGURE H1-8. PROBABILITY FACTOR $k_{P\gamma}$ FOR γ 0.90.

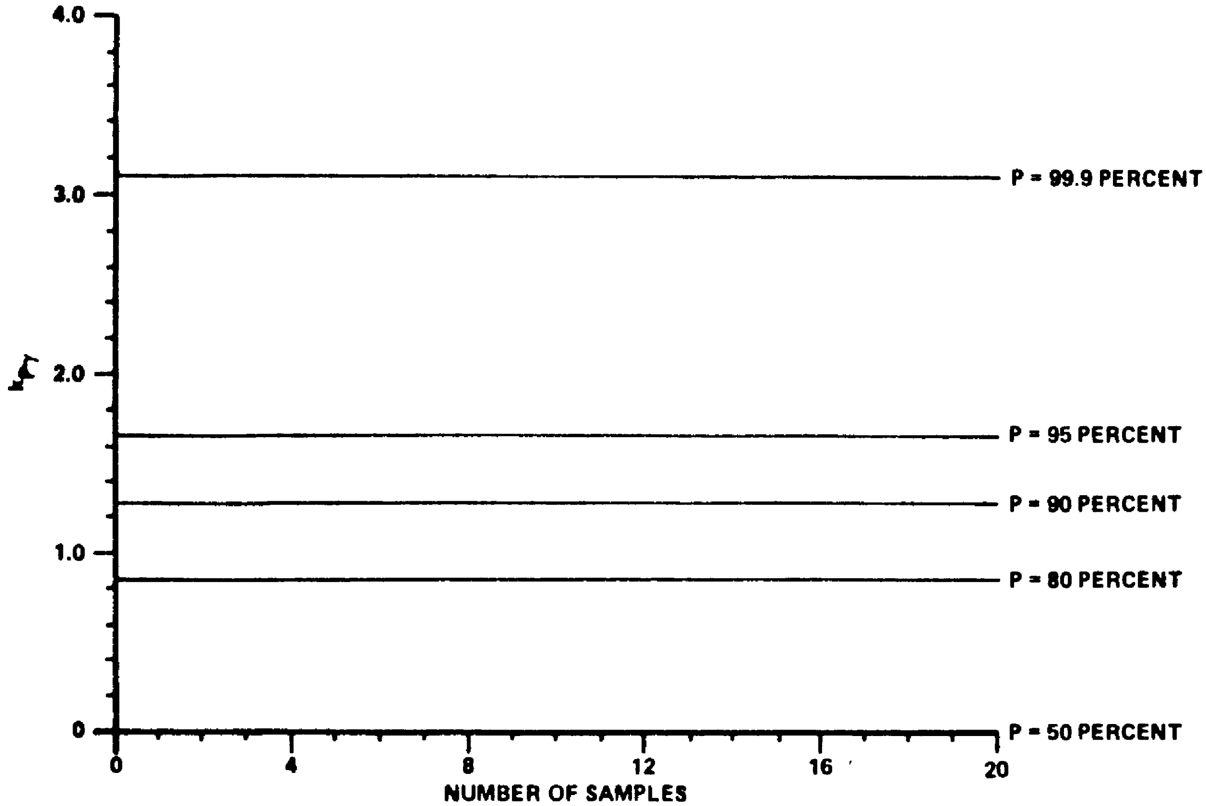

FIGURE H1-9. PROBABILITY FACTOR $k_{P\gamma}$ FOR γ = 0.50.

1.2.2.6 Example Problem 5

Using the data in Table H1-5, determine the range within which 68 percent of all points fall [3].

$\overline{\log N} = 4.3321$

$s = 0.213$

From Figure H1-6, $k_p = 1.0$

$\log N_{max} = 4.3321 + 0.213 = 4.545$ $\qquad N_{max} = 35100$ cycles

$\log N_{min} = 4.3321 - 0.213 = 4.119$ $\qquad N_{min} = 13700$ cycles

Conclude: The interval x_L to x_u is a $100 (1 - \alpha)$ percent confidence interval for P percent of the population; i.e., we may assert with 50 percent confidence the 68 percent of the population will fall within $13700 < N < 35100$ cycles.

1.2.2.7 Example Problem 6

Using the data in Table H1-5, determine the life above which 90 percent of all points in the total population will lie with a confidence of 95 percent.

Choose desired confidence level	Let $1 - \alpha = 0.95$ $\alpha = 0.05$
Choose P percent of data points which should exceed the lower limit.	Let P = 90 percent
Compute $\overline{\log x}$	$\overline{\log N} = 4.3321$

Compute s	$s = 0.213$
Find $k_{P\gamma}$ in Figure H1-7	for $P = 90$ percent, $\gamma = 95$ percent, $n = 5$ $k_{P\gamma} = 3.35$
Compute $\log N_L = \overline{\text{Log } N} - k_{P\gamma}\, s$	$\log N_L = 3.61855$ $N_L = 4155$ cycles

Conclude: We are $100\ (1 - \alpha)$ percent confident that P percent of the data points are greater than x_L'; i.e., we may assert with 95 percent confidence that 90 percent of the lives are greater than 4155 cycles.

1.3 REFERENCES

H1-1. Hald, A.: Statistical Theory with Engineering Application. John Wiley, 1952.

H1-2. Structures Manual. Convair, Fort Worth, Texas.

H1-3. Structural Methods Handbook. Temco Aircraft Corporation, Report no. 00.80.39, 1956.

H1-4. Natrella, M. G.: Experimental Statistics, National Bureau of Standards Handbook 91, August 1, 1963.

H1-5. Liu, A. F.: Statistical Variation in Fracture Toughness Data of Airframe Materials. Proceedings of the Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials, AFFDL TR 70-144, December 1969.

H1-6. Metallic Materials and Elements for Aerospace Vehicle Structures. Department of Defense, MIL-HDBK-5A, February 8, 1966.

☆ U. S. GOVERNMENT PRINTING OFFICE: 1972–745-386/Region No. 4

APPROVAL

ASTRONAUTIC STRUCTURES MANUAL
VOLUME III

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

A. A. McCOOL
Director, Structures and Propulsion Laboratory

☆U.S. GOVERNMENT PRINTING OFFICE 1977–740-049/190 REGION NO. 4